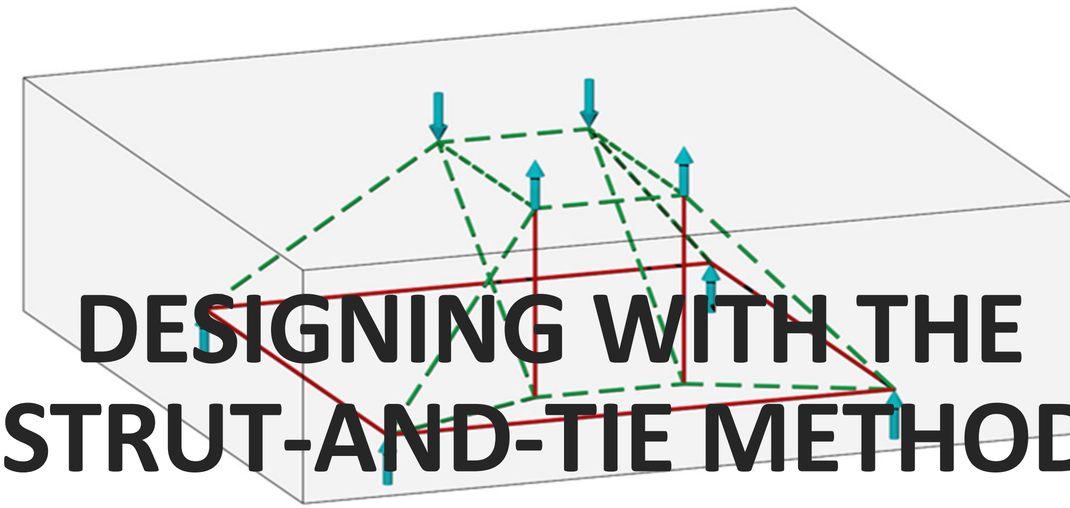


DESIGNING WITH THE STRUT-AND-TIE METHOD

Chris Williams, PhD
Assistant Professor of Civil Engineering
Purdue University

January 21, 2020

WHAT IS THE STRUT-AND-TIE METHOD (STM)?

- **Lower-bound (i.e., conservative) design method for reinforced concrete structures**
  - **Design of *D-regions* ("D" = discontinuity or disturbed)**

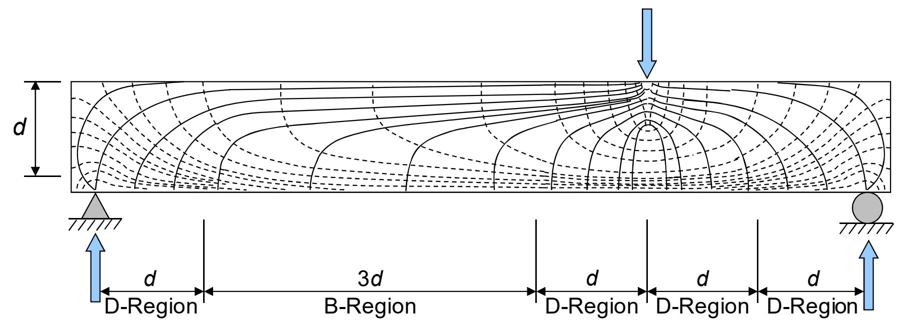


Figure: Stress trajectories within structural member

- **D-regions vs. *B-regions* ("B" = beam or Bernoulli)**

(Content adapted from Williams et al., 2012a & 2012b)

D-REGIONS VS. B-REGIONS

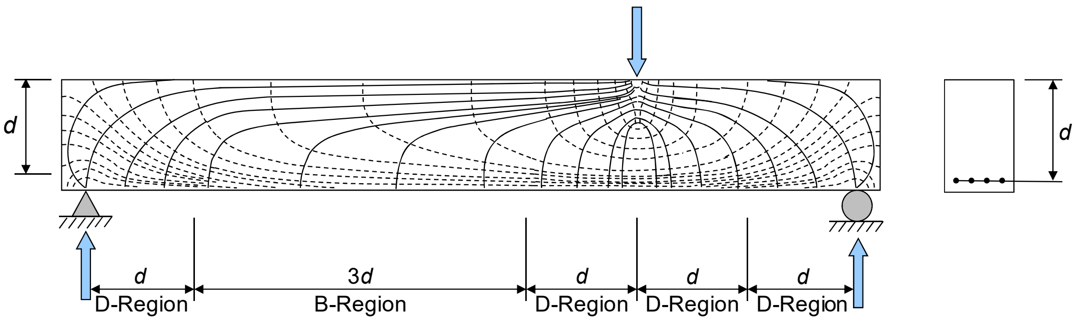
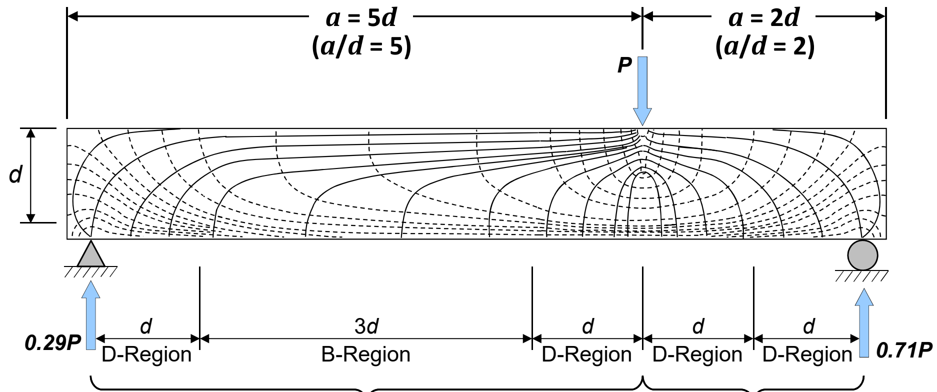


Figure: Stress trajectories within structural member

- **D-regions**
  - **Within d of load or geometric discontinuity (St. Venant's Principle)** — **Frame corner, dapped end, opening, corbel**
  - **Nonlinear distribution of strains**
- **B-regions**
  - **Linear distribution of strains**
  - **Plane sections remain plane**

3

WHEN DO YOU NEED TO USE THE STM?



Shear-span-to-depth ratio →

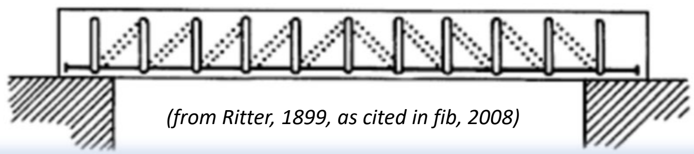
Dominated by *Sectional Behavior*
(a/d > ~2.0)
↓
Sectional Design Procedure is Valid

Dominated by *Deep Beam Behavior*
(a/d < ~2.0)
↓
Sectional Design Procedure is Invalid
∴ Use the STM

4

STRUT-AND-TIE METHOD PROVISIONS

Development of truss analogy for the behavior of reinforced concrete structures (Ritter, 1899; Mörsch, 1902)



(from Ritter, 1899, as cited in fib, 2008)

↓

Development and refinement of the STM among European researchers (Schlaich and others)

↓

5

STRUT-AND-TIE METHOD PROVISIONS

The STM introduced into AASHTO LRFD provisions in 1994

↓

The STM introduced into ACI 318 provisions in 2002

↓

Re-write of STM provisions in AASHTO LRFD 2016 Interim Revisions

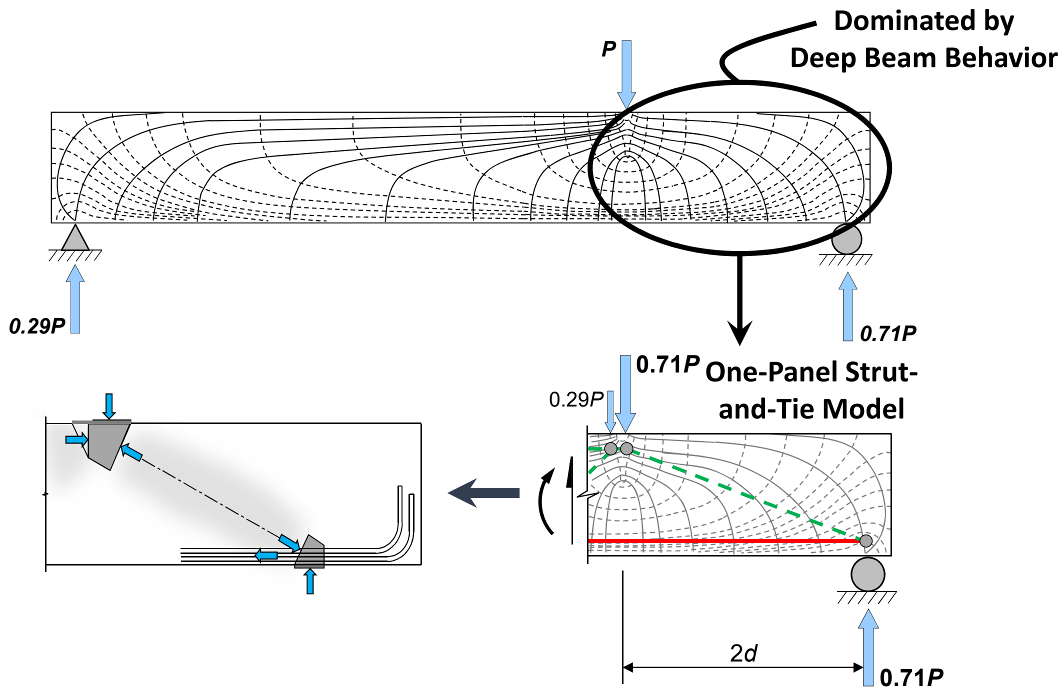
DEEP BEAM EXPERIMENTAL WORK

(Birrcher et al., 2009)

INVERTED-T EXPERIMENTAL WORK

(Larson et al., 2013)

HOW DO YOU USE THE STM?



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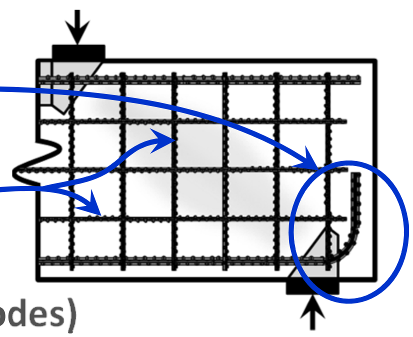
STM FUNDAMENTALS

The STM is a lower-bound (i.e., conservative) design method, provided that:

1. **Strut-and-tie model is in equilibrium with external forces (and internal equilibrium is satisfied)**
2. **Concrete element has sufficient deformation capacity to allow distribution of forces assumed by the strut-and-tie model**
   - **Key detailing requirements:**
     - ✓ **Proper anchorage of reinforcement**
     - ✓ **Distributed orthogonal reinforcement**

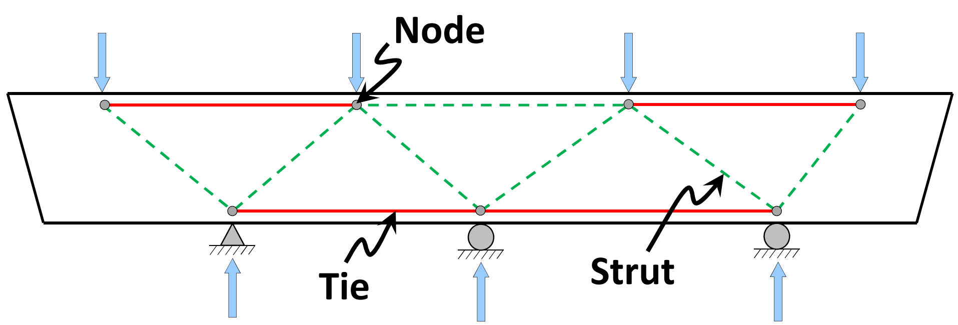


3. **Strength is sufficient (ties and nodes)**

STM FUNDAMENTALS

Three parts to every strut-and-tie model:

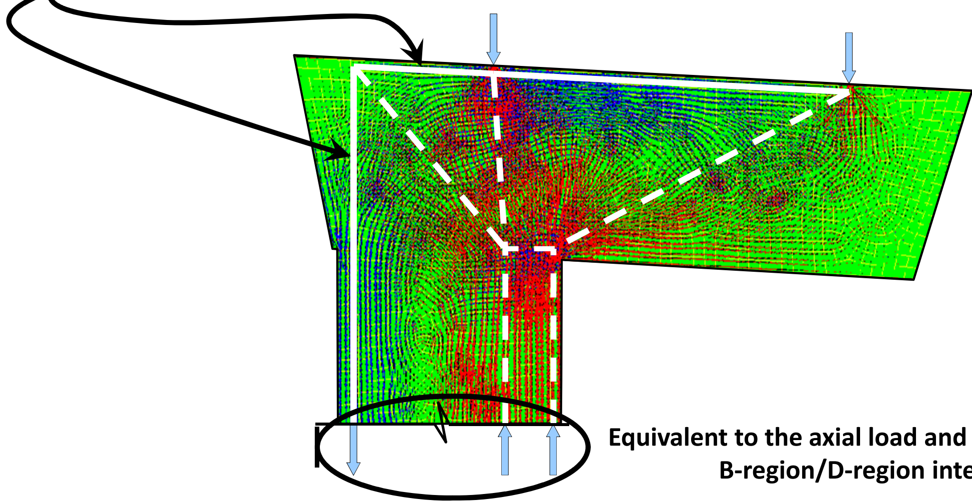
Struts **Ties** **Nodes**



STM FUNDAMENTALS

Place struts and ties according to "flow" of forces indicated by an elastic analysis

Ties must be located at the centroid of the reinforcing bars

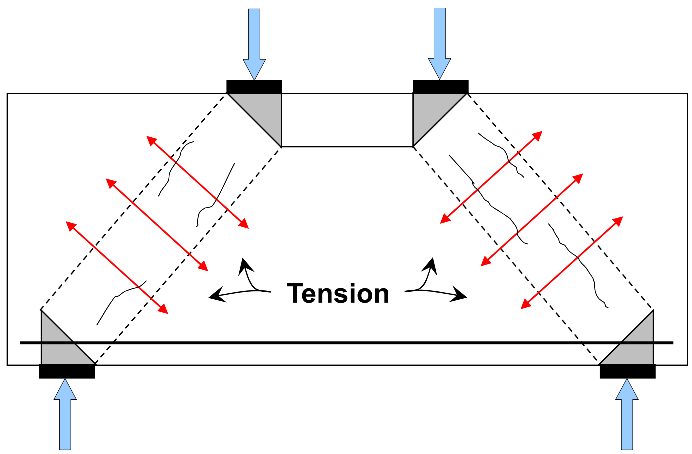


Equivalent to the axial load and moment at the B-region/D-region interface

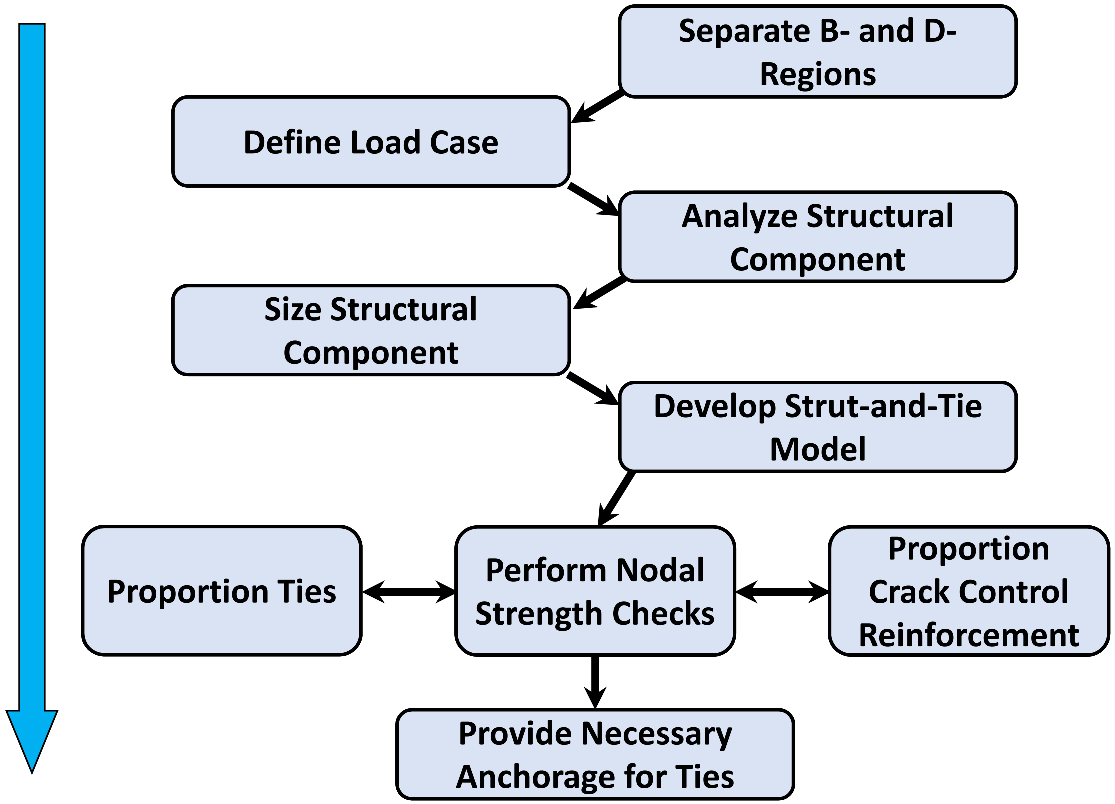
STM FUNDAMENTALS

Diagonal Tension ⊥ to Interior Struts

Transverse tension → cracking
Provide reinforcement to control cracking



STRUT-AND-TIE METHOD DESIGN PROCEDURE



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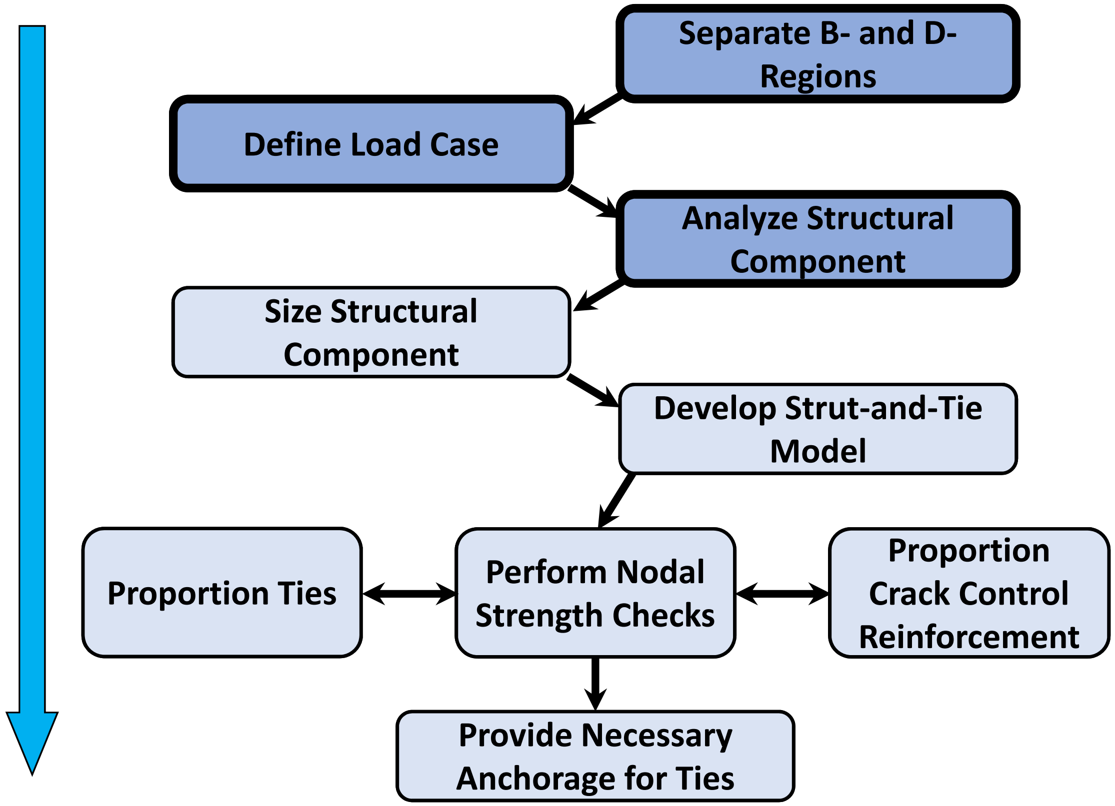
STRUT-AND-TIE METHOD DESIGN PROCEDURE



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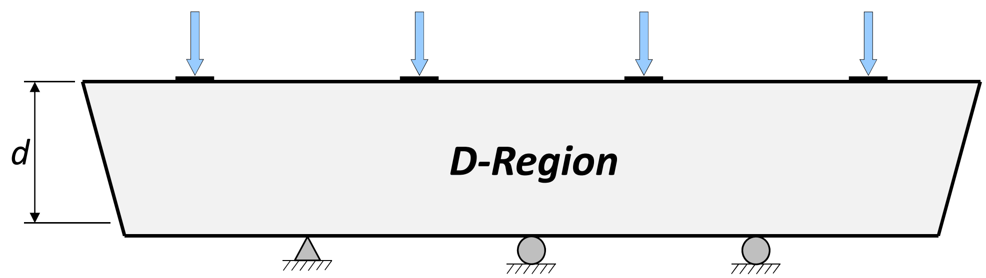
SEPARATE B- AND D-REGIONS

Apply St. Venant's Principle → *d* away from load or geometric discontinuity

Determine if region is dominated by deep beam behavior or sectional behavior



Entire member is dominated by deep beam behavior

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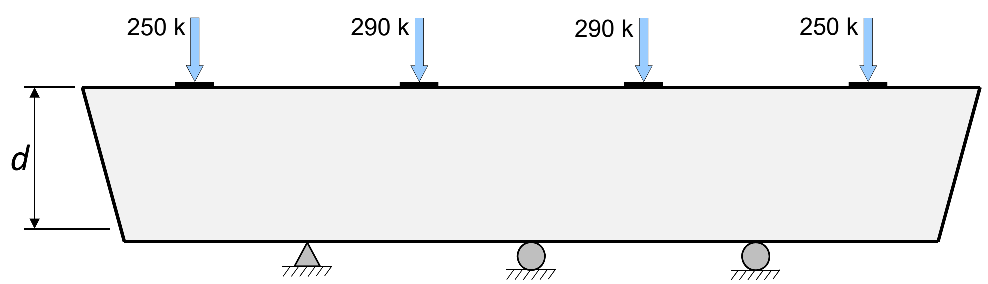
DEFINE LOAD CASE

Apply factored loads to the structural component



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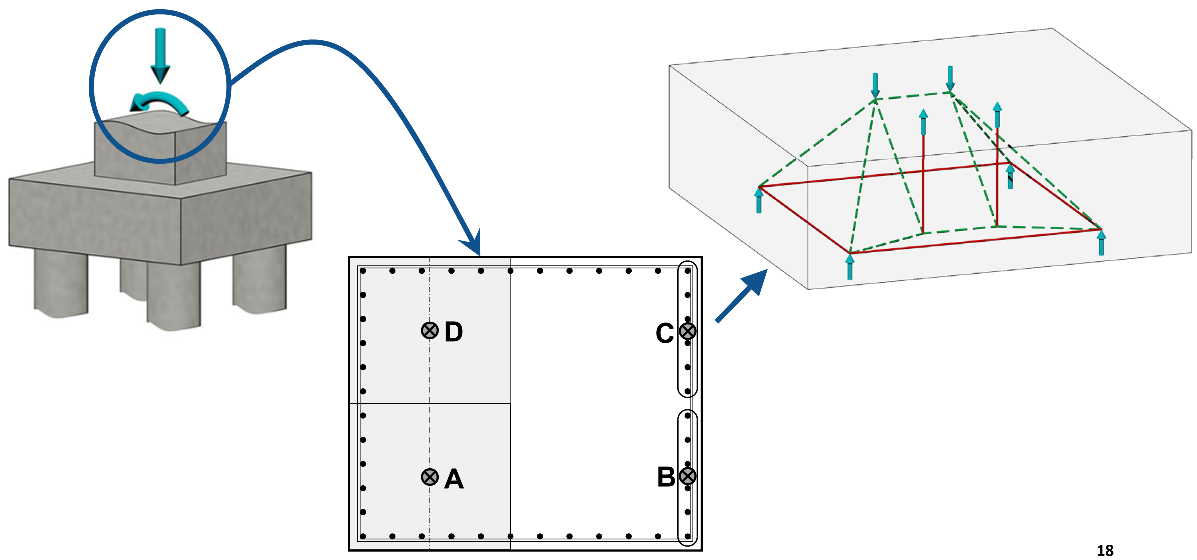
DEFINE LOAD CASE

Remember that loads must be applied to a feasible strut-and-tie model

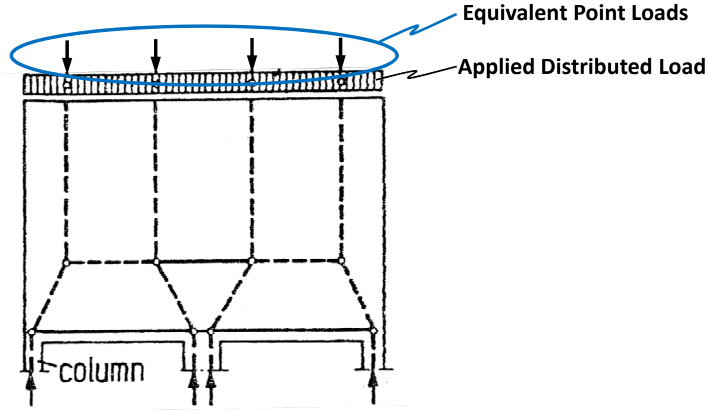
- **Moment → couple or equivalent set of forces**



DEFINE LOAD CASE

Remember that loads must be applied to a feasible strut-and-tie model

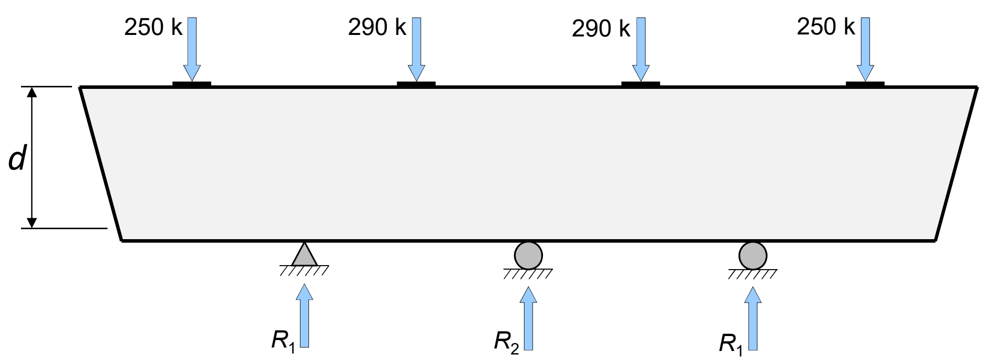
- **Distributed load → point loads**



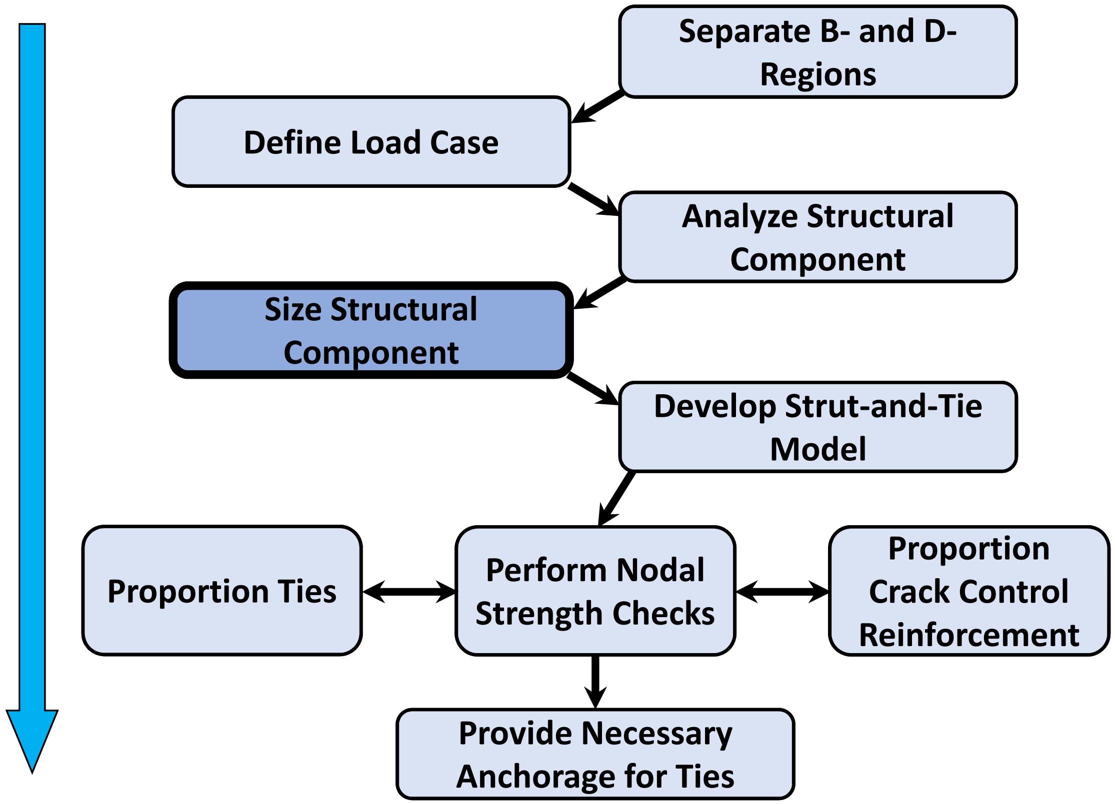
(adapted from Schlaich et al., 1987*)*

ANALYZE STRUCTURAL COMPONENT

Determine support reactions
(Keep D-region behavior in mind)



STRUT-AND-TIE METHOD DESIGN PROCEDURE



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SIZE STRUCTURAL COMPONENT

Choose geometry that reduces the risk of diagonal crack formation under service loads

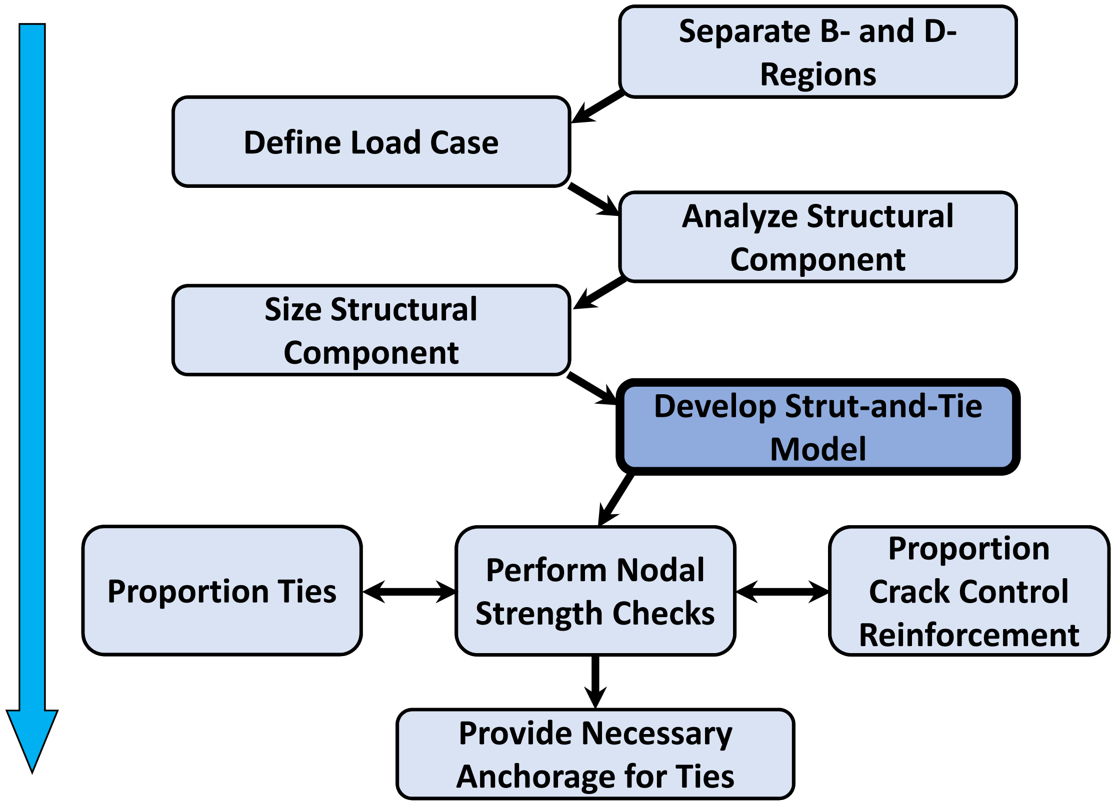
Determine dimensions so that V_{cr} for the region exceeds the maximum shear force caused by service loads (Birrcher et al., 2009)

$$V_{cr} = \left[6.5 - 3\left(\frac{a}{d}\right)\right]\sqrt{f'_c}b_w d$$

but not greater than $5\sqrt{f'_c}b_w d$ **nor less than** $2\sqrt{f'_c}b_w d$

where a = **shear span (in.)**
d = **effective depth of the member (in.)**
f'_c = **compressive strength of concrete (psi)**
b_w = **web width of the member (in.)**

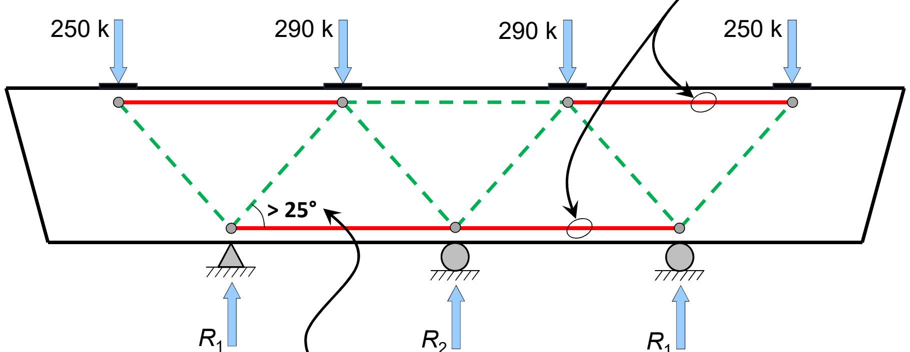
STRUT-AND-TIE METHOD DESIGN PROCEDURE



DEVELOP STRUT-AND-TIE MODEL

Place struts and ties to model the flow of forces from the loads to the supports

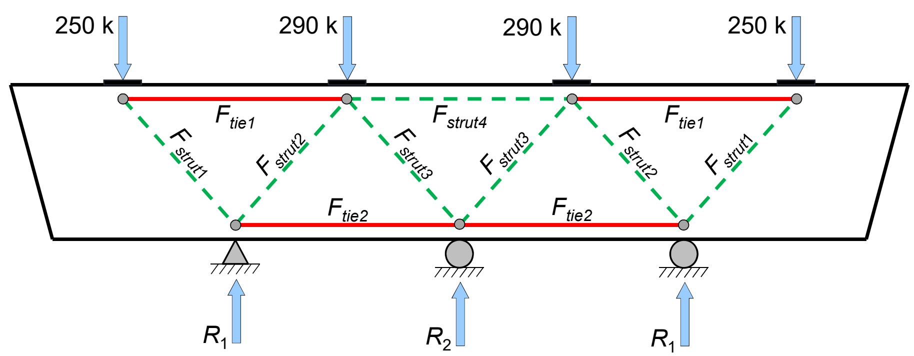
Ties must be positioned at the centroid of reinforcing bars



The angle between a strut and a tie entering the same node must be greater than 25°

DEVELOP STRUT-AND-TIE MODEL

Analyze strut-and-tie model



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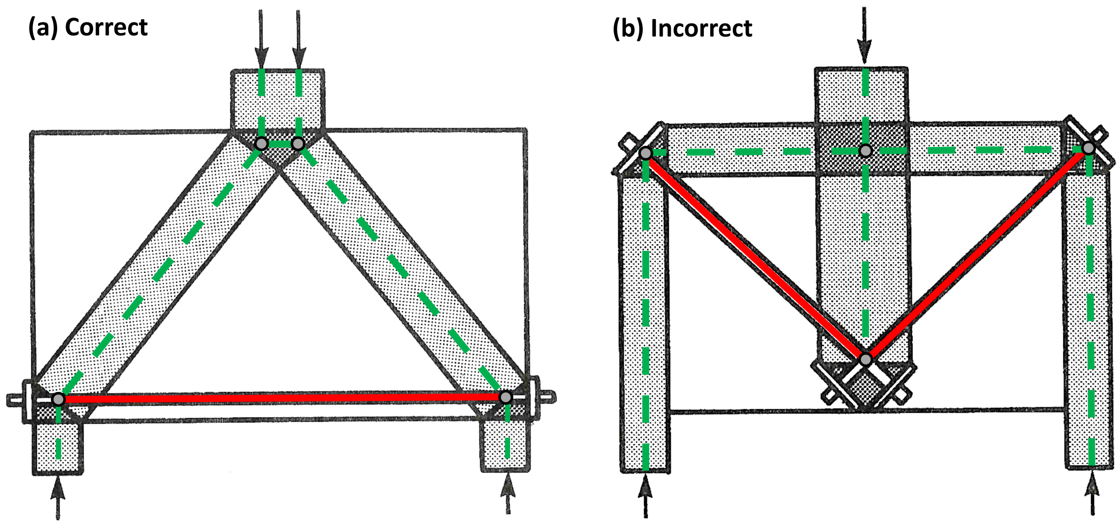
DEVELOP STRUT-AND-TIE MODEL

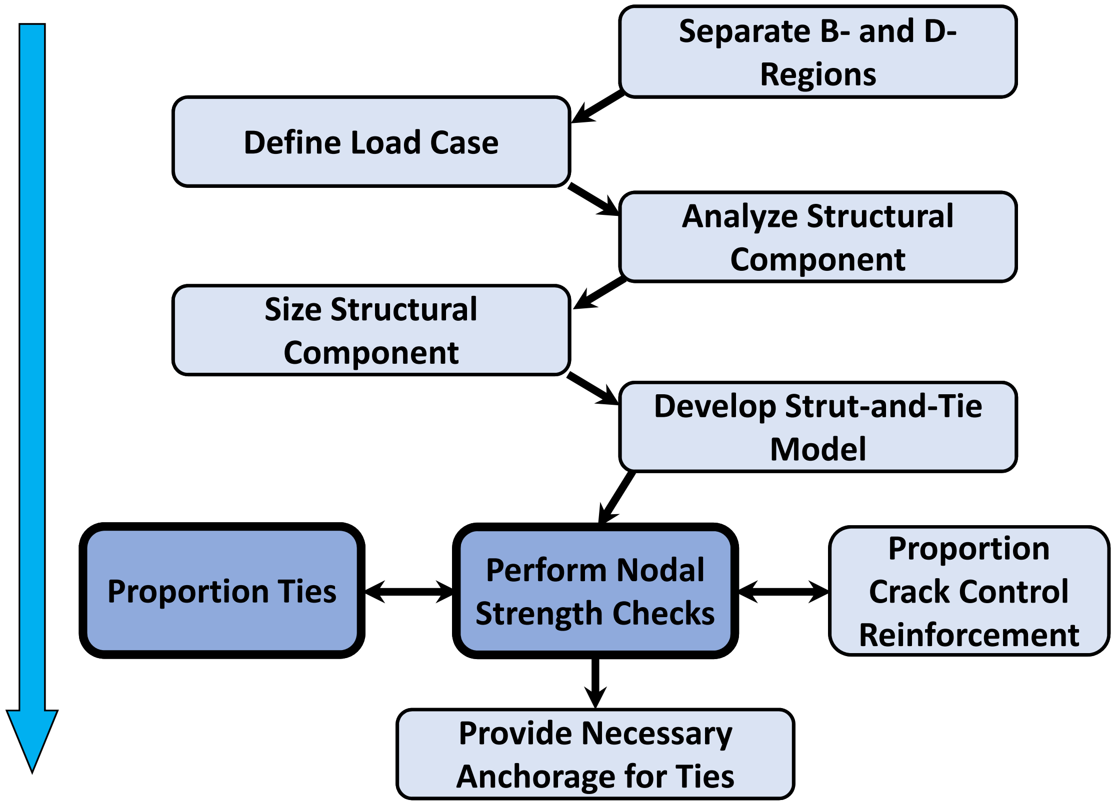
In general, model with fewest and shortest ties is the best

(a) Correct **(b) Incorrect**



(adapted from MacGregor and Wight, 2005)

STRUT-AND-TIE METHOD DESIGN PROCEDURE



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PROPORTION TIES

Determine the area of reinforcement needed to carry the calculated tie forces

$$A_{st} = \frac{P_u}{\phi f_y}$$

where A_{st} **= area of reinforcement needed to carry tie force (in.^{2})**
P_u **= factored force in tie according to the strut-and-tie model (kip)**
f_y **= yield strength of steel (ksi)**
ϕ **= resistance factor (0.90 per AASHTO LRFD)**

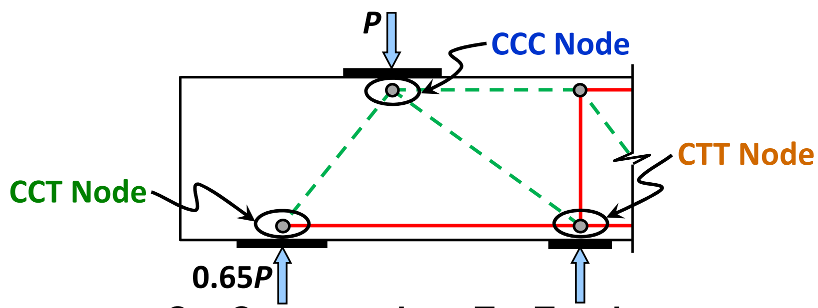
PERFORM NODAL STRENGTH CHECKS

Nodes → Typically most highly stressed regions (bottleneck of stresses)

Ensure nodal strengths are greater than the forces acting on the nodes to prevent failure

PERFORM NODAL STRENGTH CHECKS

Types of Nodes

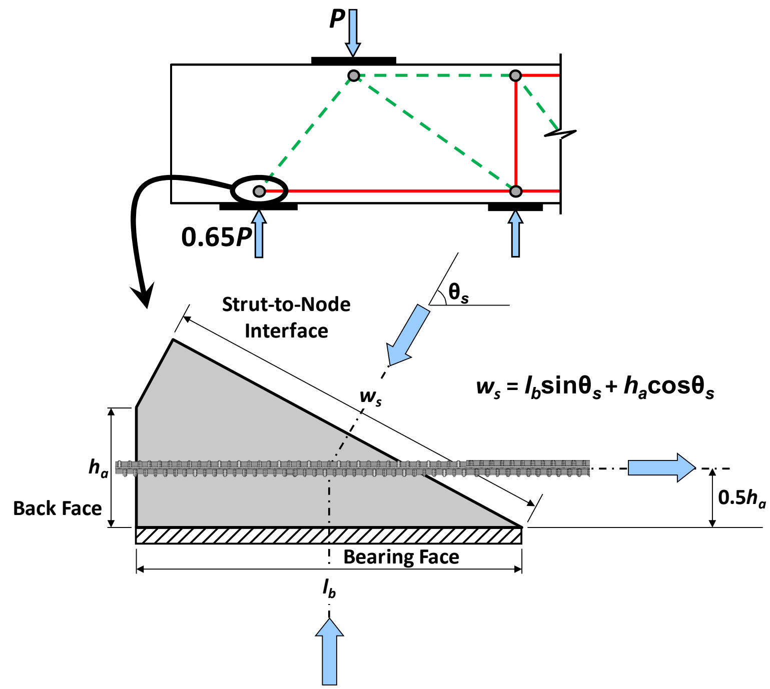


C = Compression T = Tension

- **CCC** — **Only struts intersect**
- **CCT** — **A tie intersects node in one direction**
- **CTT** — **Ties intersect node in two different directions**

PERFORM NODAL STRENGTH CHECKS

Proportioning CCT Nodes



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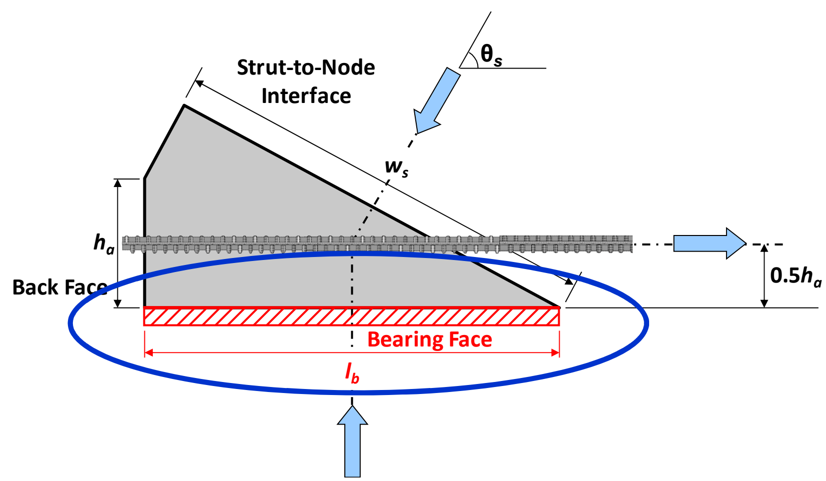
PERFORM NODAL STRENGTH CHECKS

Proportioning CCT Nodes



- **Bearing face – length is based on dimension of bearing plate**

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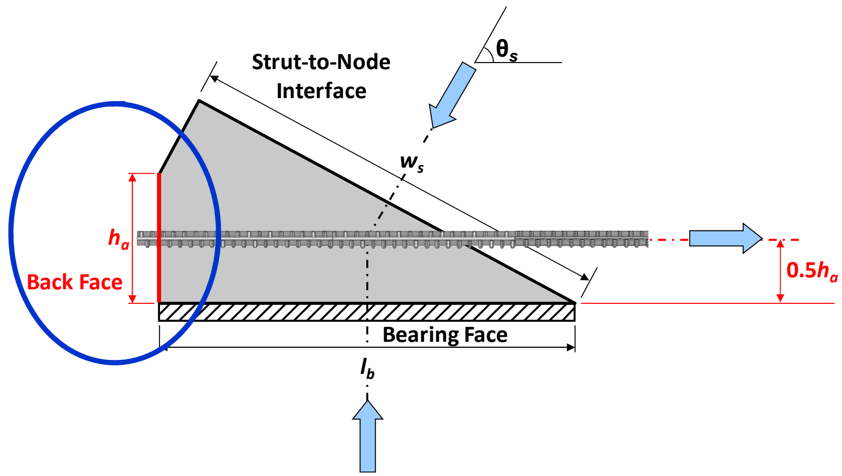
PERFORM NODAL STRENGTH CHECKS

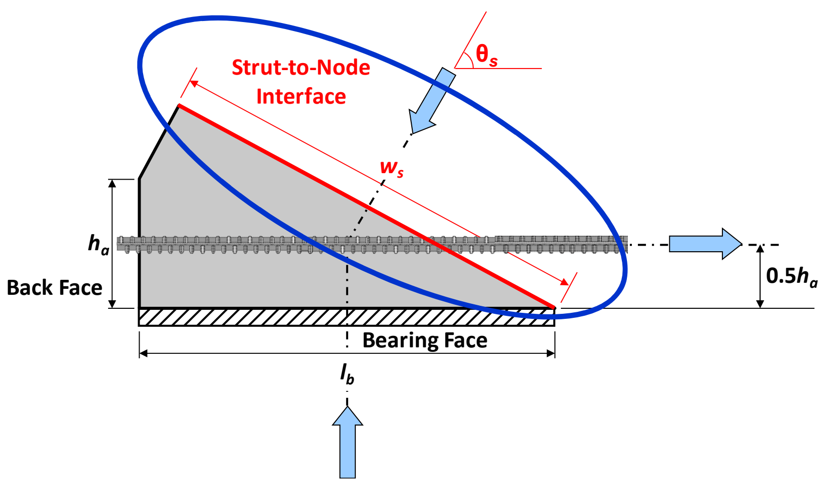
Proportioning CCT Nodes



- **Back face – h_a is based on location of longitudinal reinforcement**

PERFORM NODAL STRENGTH CHECKS

Proportioning CCT Nodes



- **Strut-to-node interface – w_s depends on lengths of bearing and back faces as well as angle of diagonal strut**

$$w_s = l_b \sin\theta_s + h_a \cos\theta_s$$

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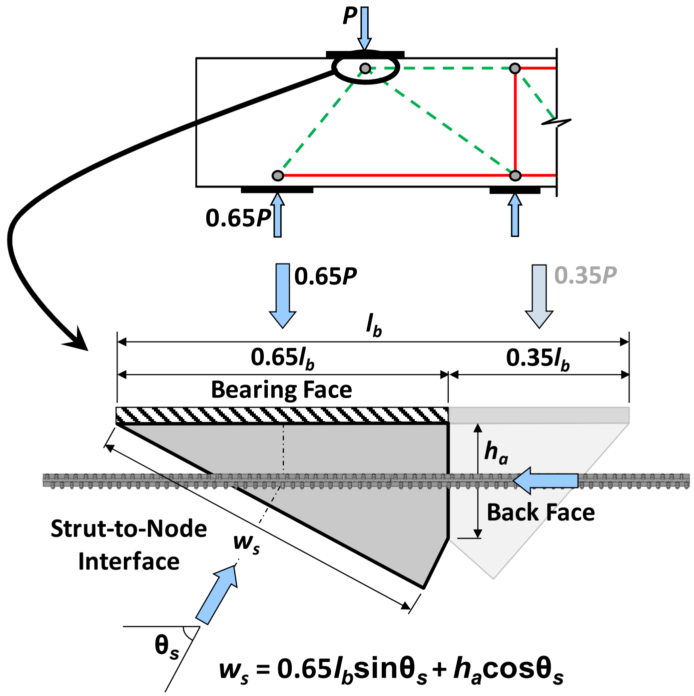
PERFORM NODAL STRENGTH CHECKS

Proportioning CCC Nodes



$$w_s = 0.65l_b\sin\theta_s + h_a\cos\theta_s$$

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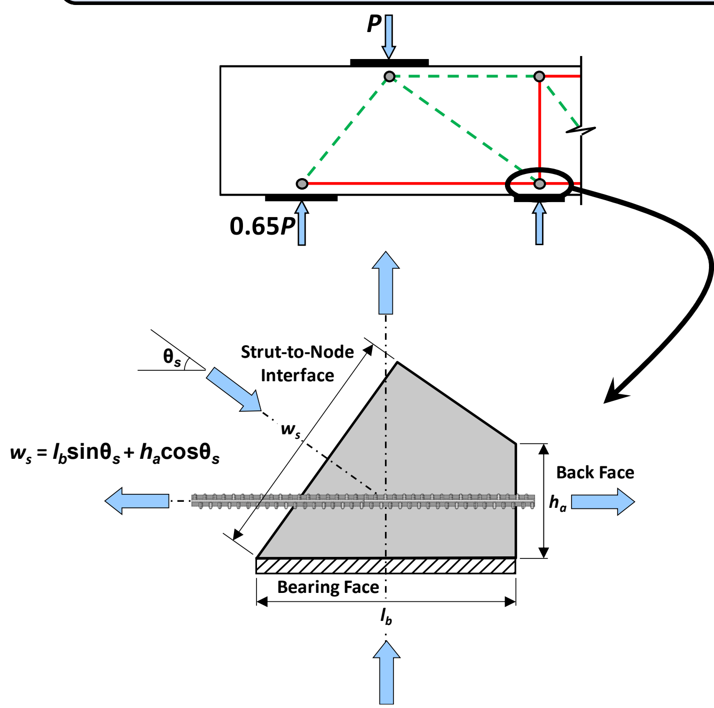
PERFORM NODAL STRENGTH CHECKS

Proportioning CTT Nodes



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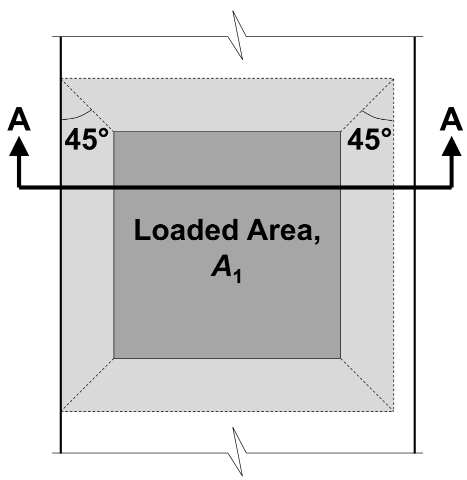
PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

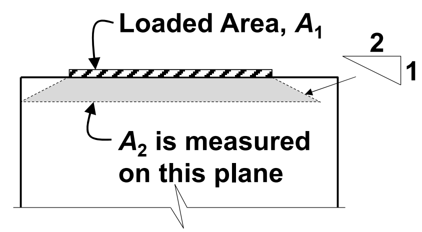
Step 1 – Calculate confinement modification factor, *m*

$$m = \sqrt{A_2 / A_1} \leq 2.0$$



Plan View

***m*-factor can be applied to all faces of the node**



Section A-A through Member

(AASHTO LRFD, 2017)

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PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

Step 2 – Determine concrete efficiency factor, ν, for node face under consideration

	Node Type		
Face	***CCC***	***CCT***	***CTT***
Bearing Face	$\mathbf{0.85}$	$\mathbf{0.70}$	$\mathbf{0.85 - f'_c/20\ ksi}$ $\mathbf{0.45 \le \nu \le 0.65}$
Back Face			
Strut-to-Node Interface	$\mathbf{0.85 - f'_c/20\ ksi}$ $\mathbf{0.45 \le \nu \le 0.65}$	$\mathbf{0.85 - f'_c/20\ ksi}$ $\mathbf{0.45 \le \nu \le 0.65}$	

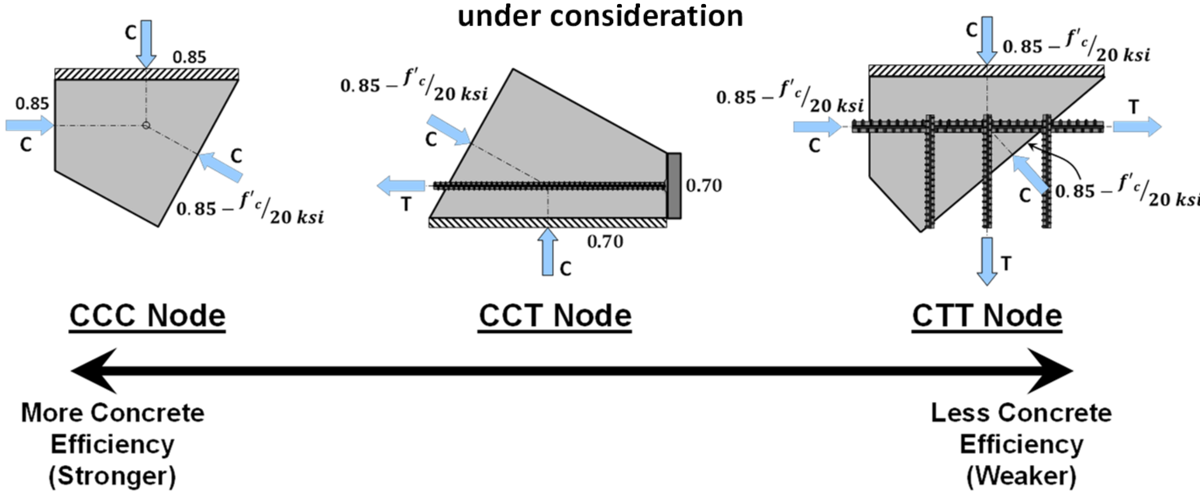
(AASHTO LRFD, 2017)

If the web crack control reinforcement requirement is not satisfied, use ν = 0.45 for the strut-to-node interface

PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

Step 2 – Determine concrete efficiency factor, ν, for node face under consideration



If the web crack control reinforcement requirement is not satisfied, use ν = 0.45 for the strut-to-node interface

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PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

Step 3 – Calculate the design strength of the node face, ϕP_n

$$\phi \cdot P_n = \phi \cdot f_{cu} \cdot A_{cn}$$

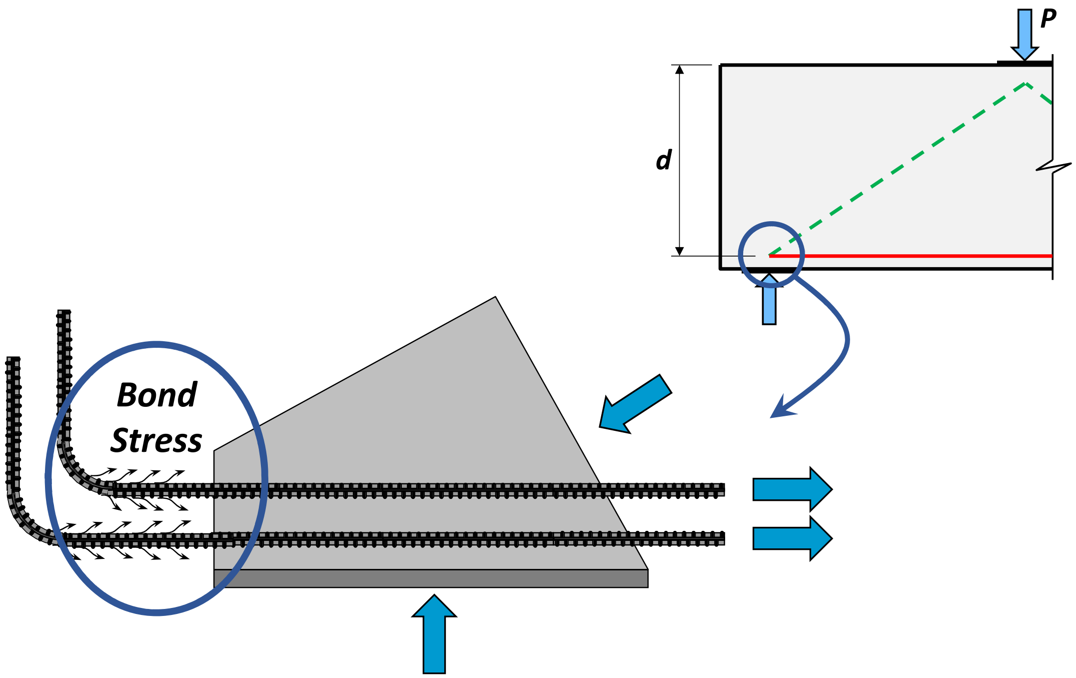
$$f_{cu} = m \cdot \nu \cdot f'_c$$

where f_{cu} = limiting compressive stress (ksi)
ϕ = resistance factor for compression in strut-and-tie models (0.70 per AASHTO LRFD)
A_{cn} = effective cross-sectional area of the node face ($in.^2$)

Ensure the design strength, ϕP_n, is greater than or equal to the factored force, P_u, acting on the node face:

$$\phi P_n \geq P_u$$

PERFORM NODAL STRENGTH CHECKS



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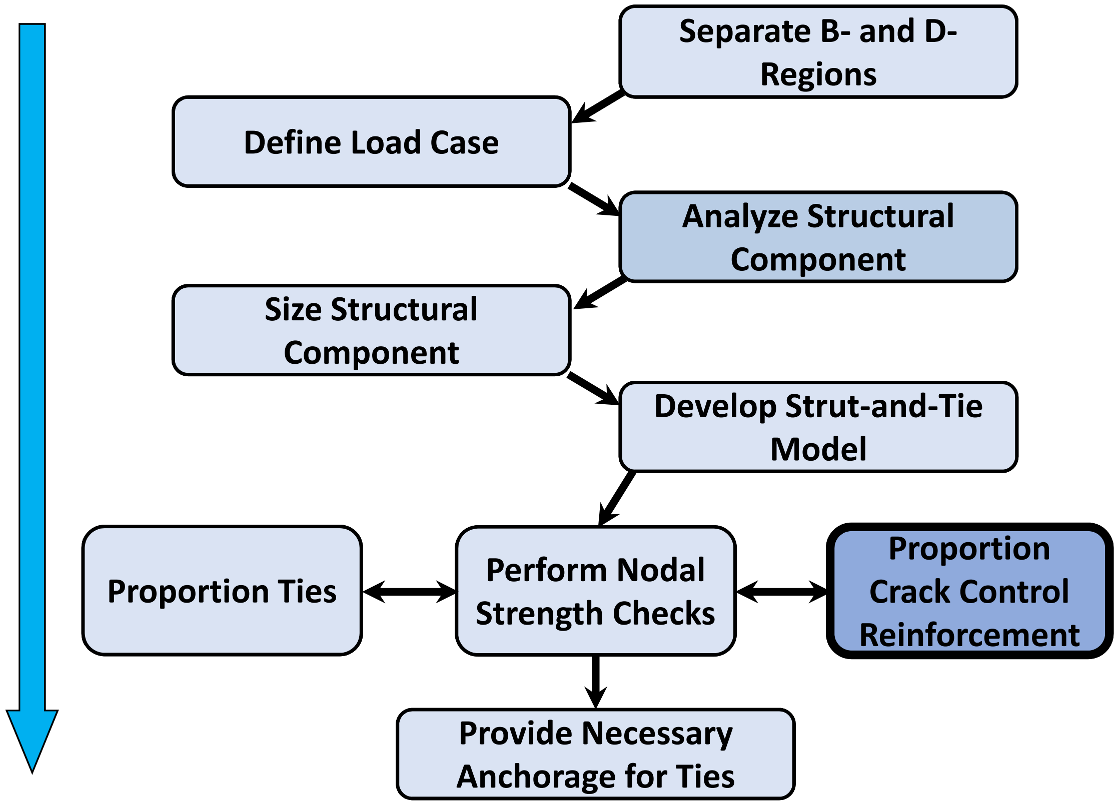
STRUT-AND-TIE METHOD DESIGN PROCEDURE



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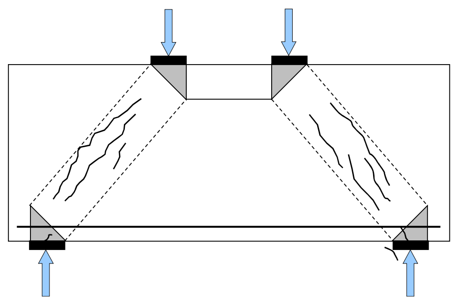
PROPORTION CRACK CONTROL REINFORCEMENT

Provide distributed orthogonal reinforcement that can:

- **Carry diagonal tensile stress transverse to interior struts**
- **Restrain bursting cracks caused by this tensile stress**



- **Increase ductility by allowing redistribution of stresses**
- **Prevent premature strut failure**

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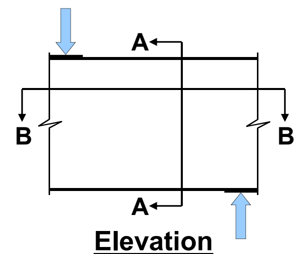
PROPORTION CRACK CONTROL REINFORCEMENT

Provide 0.3% reinforcement in each orthogonal direction

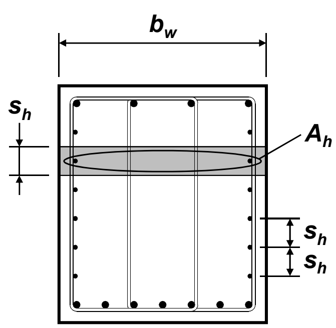
- **Evenly space reinforcement as shown**

$$\frac{A_v}{b_w s_v} \geq 0.003 \qquad \frac{A_h}{b_w s_h} \geq 0.003$$

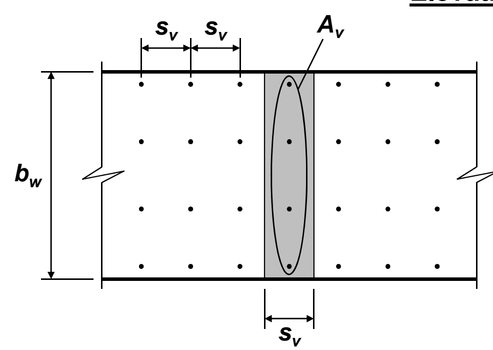
s_v and s_h shall not exceed $d/4$ or 12 in.



Elevation



Section A-A



Section B-B

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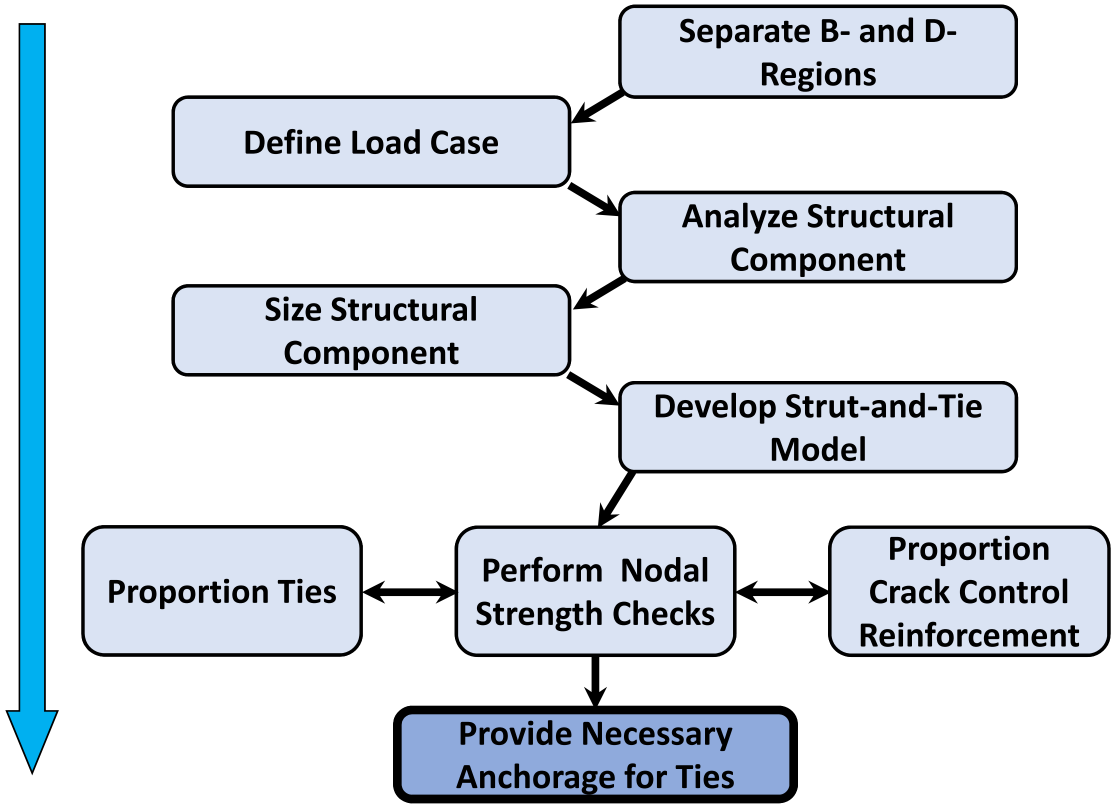
STRUT-AND-TIE METHOD DESIGN PROCEDURE



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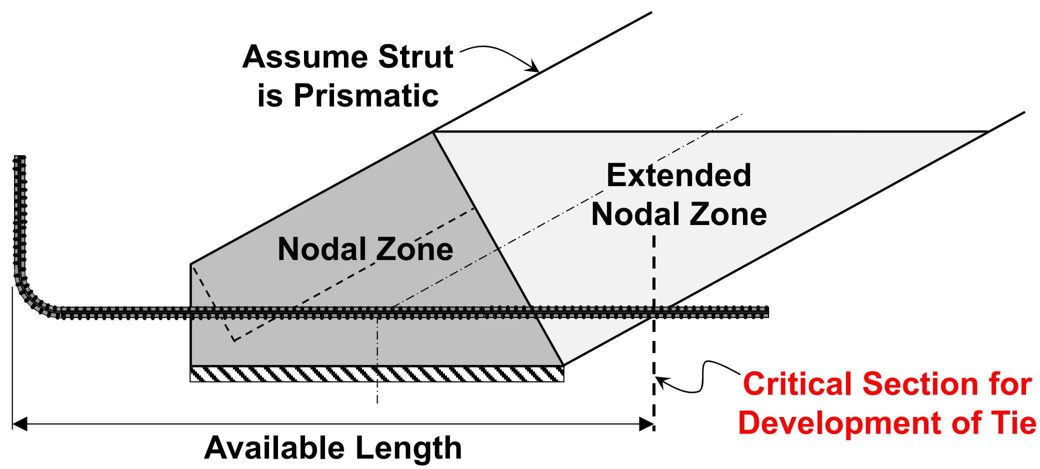
PROVIDE NECESSARY ANCHORAGE FOR TIES

Reinforcement must be fully developed at the point where the centroid of the bars exits the *extended nodal zone*



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KEY POINTS FOR STM DESIGN

1. **At the interface of a B-region and a D-region, ensure the internal forces and moment within the B-region is applied correctly to the D-region**

2. **A tie must be located at the centroid of the reinforcement that carries the tie force**

3. **The angle between a strut and a tie entering the same node must be greater than 25°**

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KEY POINTS FOR STM DESIGN

4. **The strut-and-tie model must be in external and internal equilibrium**

5. **Ensure proper reinforcement detailing → a strut-and-tie model is only as good as the details**

KEY POINTS FOR STM DESIGN

6. **Make reasonable, conservative assumptions and simplifications when necessary**

7. **Make everything as simple as possible, but not simpler**

Introduction to STEP (Strut-and-Tie Evaluation Program)

INTRODUCTION TO STEP

Aids with the Design of Substructure Components:

- **Multi-Column Bent Caps**
- **Straddle Bent Caps**
- **Integral and Semi-Integral End Bent Caps**

BEFORE USING STEP

Understand Limitations and Assumptions

INSTRUCTIONS

To use STEP, designers should begin by entering required user inputs on the Inputs worksheet in any cell with a yellow background. Cells that do not have a yellow background should not be edited. When inputting values, ensure that appropriate units are selected. Units in purple text can be changed, while units in black text cannot be changed. It is not necessary to format the Inputs worksheet. The sheet will be automatically reformatted when the program is run. Once the designer has correctly input all required information, the "Run Program" button should be clicked to start the STM design procedure. If the program has been run previously and not yet reset, a message will appear informing the user that running the program will cause the previous results to be deleted. The user should click “Yes” to allow the program to run. Another message will appear when the program run is complete. The "Reset Program" button deletes the results of a previous run of the program (i.e., all worksheets other than this Instructions sheet and the Inputs sheet will be deleted when the button is clicked), and the Inputs sheet is returned to its default state. The "Rerun Program" button should only be used after running the program once. Detailed information about when to use the "Rerun Program" button is included in the STEP Guidebook.

For more information about how to use STEP, reference the STEP Guidebook.

DISCLAIMER

STEP is a tool developed to aid engineers with implementing the strut-and-tie method for the design of bridge substructure components. The engineer should be familiar with the limitations and the assumptions of the program prior to using it as a design tool. It is the responsibility of the engineer to ensure the accuracy of the results and interpret whether the design is safe and serviceable. The developers of STEP made every effort to create a program that will generate viable strut-and-tie models and accurately perform design procedures for most typical nonprestressed multi-column bent caps and straddle bent caps . Nevertheless, some particular design situations may have been inadvertently overlooked, which may cause the program to output erroneous results. Ultimately, the engineer is responsible for verifying the viability and accuracy of the strut-and-tie model and validating all results output by the program.

Limitations

1. The program will perform an elastic continuous beam analysis to determine the magnitude of support reactions if support reaction values are not input by the user. Determining the support reactions of members dominated by D-regions using alternative methods may be more accurate. The user can input support reaction values calculated by an alternative method into the program, and the forces in the strut-and-tie model will be computed based on these values.
2. The top and bottom chords of the strut-and-tie model are located at constant depths along the length of the member. In other words, the location of the top and bottom chords does not change along the length of the bent cap. If there is a negative moment region, the top chord will be located at the centroid of the top longitudinal reinforcement. Otherwise, if there is no negative moment, the location of the top chord will be based on an optimized design.
3. The amount of top and bottom longitudinal reinforcement is constant along the length of the member

Assumptions

1. The strength of a back face is only checked if compression acts on the face. Article 5.8.2.5.3b of AASHTO LRFD states that “[b]ond stresses resulting from the force in a developed tie[…]need not be applied to the back face of the CCT node.” However, this phenomenon may also occur at CTT nodes. (See Node P of Example 1 in “Strut-and-Tie Model Design Examples for Bridges.” The horizontal component of the diagonal strut must transfer to the longitudinal reinforcement as a bond stress, changing the force in the horizontal tie at the node.) The program considers this observation and only checks the strength of a back face when it is subjected to direct compressive stresses.
2. Because bent caps do not fall under the category of slabs or footings, horizontal and vertical crack control reinforcement in accordance with Article 5.8.2.6 of AASHTO LRFD should be provided. Therefore, the efficiency factors in Table 5.8.2.5.3a-1 of AASHTO LRFD are used for all nodal strength calculations.
3. The width of each vertical tie is assumed to equal the smaller length of the two adjacent “truss panels” of the strut-and-tie model and is centered on the tie

Review the STEP Guidebook

USER INPUTS

Before inputting information on this tab, see **Instructions** tab.

Notes:
-Click on any cell with ***BLUE*** *text for additional information.*
-Units in ***PURPLE*** *text can be changed. Units in* ***BLACK*** *text cannot be changed. Input standard bar size (e.g., input "5" for a No. 5 bar).*

Reset Program — Deletes all existing output sheets and clears *Inputs* sheet.

Run Program — Deletes all existing output sheets and runs program.

Rerun Program — Creates strut-and-tie model based on the "Nodes" table on the *Nodes and Members* sheet (see Guidebook Section 4.2.9).

Geometric Properties		
Length		ft
Height		ft
b_w		ft

Stirrups		
f_y	60	ksi
Bar Size (no.)		
No. of Legs		

Load Factor		
Self-Weight Factor		

Horizontal Crack Control Reinforcement		
Bar Size (no.)		
# Bars/s_h	2	

Concrete Material Properties		
f'_c		ksi
w_c	150	pcf

Longitudinal Reinforcement

Bottom Reinforcement			
f_y		60	ksi
No. of Layers			
Development Length Straight			in.
Development Length Hook			in.
Clear Cover at End of Bar		2	in.
Layer	Location from Bottom (in.)	Number of Bars	Bar Size (no.)
1			
2			
3			
4			

Top Reinforcement			
f_y		60	ksi
No. of Layers			
Development Length Straight			in.
Development Length Hook			in.
Clear Cover at End of Bar		2	in.
Layer	Location from Bottom (in.)	Number of Bars	Bar Size (no.)
1			
2			
3			
4			

Factored Applied Loads

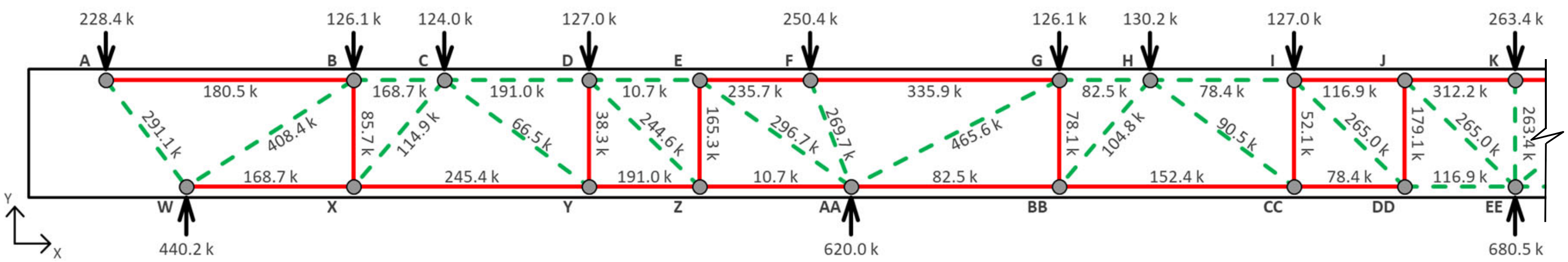
Location from Left	Load Value	Width of Loaded Area	Length of Loaded Area
ft	kip	in.	in.

Reactions

Location from Left	Support Reactions	Width of Bearing Area	Length of Bearing Area
ft	kip	in.	in.

FUNCTIONALITY OF STEP

Automatically Develops Strut-and-Tie Model



Performs Strength Checks and Other STM Design Procedures

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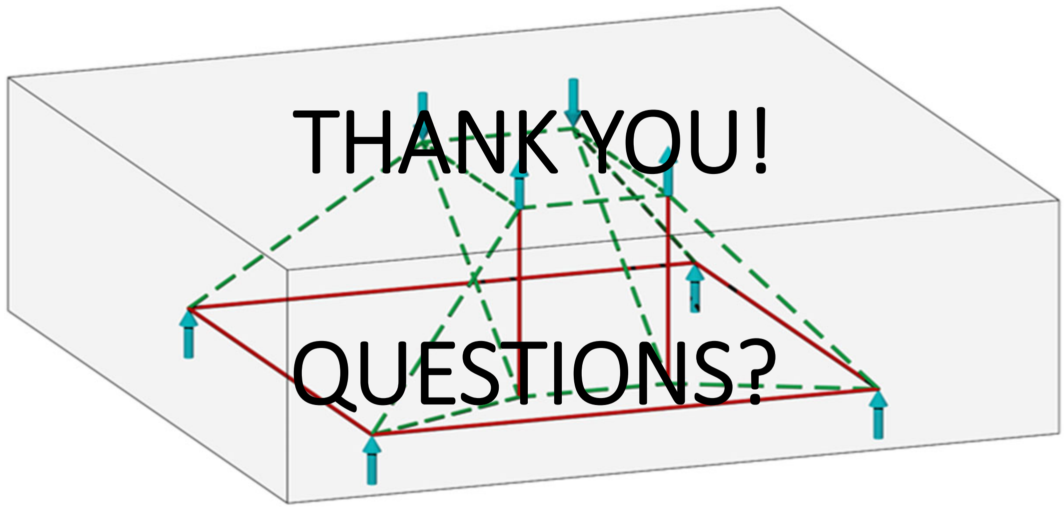

THANK YOU!

QUESTIONS?

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